

**Optimization for Sustainability of Integrated Ecological-Economic Model System
of Planet**

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Abstract

In order to successfully understand and attempt to increase the sustainability of a system, the dynamic relationship between the many dimensions of sustainability need to be understood. Through the use of mathematical models, future scenarios can be studied which can be used to possibly develop policy guidelines. The model used in this article is an integrated ecological-economic model which incorporates a simplified ecological food web, a macro-economic framework, and a rudimentary legal system. By altering certain constraints within the model, the scenarios of a population explosion and an increase in per capita consumption can be evaluated. Dynamic optimization, specifically discretized non-linear programming, is then used to develop regulatory policies using Fisher Information as a measure of sustainability.

1. Introduction:

An increase in human growth and activity has placed an enormous stress on Earth's natural resources.⁶ The realization that this detrimental behavior cannot be sustained forever has led to a significant increase in sustainability research and has now become a new focus of analysis. The Brundtland Report (1987) defines sustainability as "development that meets the needs of the present without compromising the ability of the future generations to meet their own needs".⁷ Sustainability is not a specific goal, but rather a path through time with many influencing dimensions (e.g. economic, technological, ecological, social, etc.). If the path of a system stays within certain prescribed limits, the system is said to be sustainable. The goal of sustainability management, then, is to promote the stability of the economic and human components of the system while reinforcing ecological aspects as well.² Numerous influencing factors as well as projection of long term effects make the study of sustainability so complex.

In order to understand the relationship between these dimensions of sustainability, mathematical models are used. These models are able to produce qualitative data which can be useful to policy makers to evaluate long term strategies to increase the sustainability of an ecosystem without the risk or cost of using a real ecosystem. The integrated ecological-economic model (Fig.1) studied by Kotecha et al. (2013) is used in this work to simulate a simplified, yet realistic model of the ecosystem.⁵ Previous models (Shastri et al. (2008)) assumed the system was open to energy.⁹ However, this model includes a non-renewable energy source and an energy producer, and also incorporates the use of biomass to produce energy. The scenarios of human population explosion and increase in per capita consumption are two possible scenarios expected in the future and can be simulated and studied using this model. Numerical optimization is then applied to the model to develop time-dependent management strategies in order to increase the lifetime of dying compartments seen in the two scenarios. The paper is organized as follows: section 2 explains the model used in this work which is a small representation of the dynamic behavior of the Earth. Section 3 compares the

two proposed scenarios with a base case scenario. Section 4 explains the methodology of optimization while section 5 summarizes the results of policy development.

2. Model Structure:

Fig.1 shows the integrated ecological-economic model used in the simulation. This is a compartmental model composed of a four trophic level food chain consisting of three primary producers (P1, P2, P3), three herbivores (H1, H2, H3), two carnivores (C1, C2), and humans as the top omnivore. The model is closed to mass therefore representing a simplified world. The model also consists of two resource pools; the resource pool (RP) which represents all natural resources (air, water, sunlight, etc.) and the inaccessible resource pool (IRP) which represents biologically inaccessible resources due to industrial production.¹ The energy source (ES) represents a finite non-renewable energy source. The energy producer (EP) uses the energy from ES to produce a usable form of energy which is supplied to human households (HH) and the industrial sector (IS). EP can also use mass from P1 to produce energy which represents the production of energy using biomass.

The compartments can be separated into a domestic and a non-domestic sector based on human control over the compartments. The domesticated group, P1 and H1, are private property and represent agricultural and livestock activities. The compartments in the non-domesticated group represent species that are hunted, gathered, and not consumed by humans. The domestic sector is a part of the economy while the non-domestic sector is not. P1, H1, HH, IS, EP, and ES are involved in the economy. The four industries in the model which humans can choose to work are P1, H1, IS, and EP, and wages are set by the IS based on demand and the population. The model also incorporates a rudimentary legal system. There are grazing rights which allow H1 to access P2, and fences which limit the access of H2 to P1 and similarly C1 to H1.

Some of the important features of the model include: a food web model based on trophic levels with a decrease in number of species for higher levels, species food preferences, presence of humans with a simple economic and legal system, a generic energy source, the use of biomass to produce energy, and a discharge fee for the IS. The model is characterized by 98 constant parameters, 19 time dependent state variables, and 61 model outputs. Some of the compartments represent state variables where the remaining state variables are deficits or the human population. The deficits refer to the accumulated difference between the demand for a good and the actual amount delivered for a given time step. The state variables are all in terms of mass with the exception of human population. Three general types of equations are used to describe the model: basic food web model equations, macroeconomic model equations, as well

as other algebraic equations related to factors within the model. More details of the model can be found in Kotecha et al. (2013) and its supporting documentation.⁵

3. Simulated Scenarios:

3.1. Base Case:

There are two cases to be evaluated in the model simulation; with or without the use of biofuels as a source of energy. The base case simulation, where there is no increase in consumption or population explosion, offers insights into the model and can be used to study the stability of the model. The effects of using biofuel as a source of energy can also be investigated. Using P1 as a source for energy production will slow the depletion of ES; however, the dynamics of other compartments within the model may not be necessarily intuitive due to the integrated aspect of the model. The dynamics of various ecological and economic compartments are shown below.

The dynamics of compartments P1, H1, and C1 are shown in Fig.2. As it can be seen, P1 decreases more rapidly with the use of biofuels due to the fact that some of P1 is being used for energy production. H1 stays at about the same level with and without the use of biofuels and C1 decreases more rapidly with the use of biofuels due to the lower levels of H2, caused by the lower levels of P1. It can be noted that none of the species reach extinction with or without the use of biofuels.

Fig.3 shows the dynamics of the population and the mass of the human household compartment. It can be seen that the population is constant throughout the simulation and is the same with or without the use of biofuels. However, the mass of the human household compartment increases over the simulation, though the increase is not as great with the use of biofuels. This indicates that in the future, a person will consume more resources but the per capita mass will be lower with the use of biofuels.

The dynamics of the energy source is also shown in Fig.3. It can be seen that the energy source is not depleted as quickly with the use of biofuels since some of the energy demand is being met with the use of biofuels. From this simulation it can be concluded that none of the compartments reach extinction and the model is therefore stable for the simulation time.

3.2. Human Population Explosion Scenario:

According to the Census Bureau, the world population has increased from about 2.5 billion in 1950 to about 7 billion in 2013.¹¹ This significant increase has placed a severe stress on natural resources and poses a significant threat to the environment and the sustainability of the ecosystem. For the future, the mortality rate is expected to drop due to better health care, yet the birth rate is also expected to decrease due to better education and birth control awareness

(especially in developing countries). It is widely believed that in the next 50 to 100 years the population will peak and level off to about twice today's size. A steady drop is then expected due to an aging population and a decrease in fertility rates.^{5,10} This is accepted as the most likely scenario for population growth in the future and is simulated by varying the mortality rate and per capita births. The resulting population dynamic is shown in Fig.4. It can be seen that the population is the same with or without the use of biofuels and the population reaches a maximum at about the 4500th week or around the 87th year in the simulation which is consistent with the estimated trend in population.

The mass fluctuations for some ecological compartments with and without the use of biofuel as a source of energy are shown in Fig.5. It can be seen from this figure that the amount of P1 decreases faster with the use of biofuels, due to the fact that some of P1 is being used to produce energy. P3 levels increase due to an increase in RP which then causes an increase in H3. The decrease in P2 can be attributed to the growth of H3. There is a significant decrease in the levels of H1 with the use of biofuels due to the decreased levels of both P1 and P2. There is an increase in H2 due to the decreased level of C1 caused by the low levels of H1; hence the mass flow of H2 to C1 decreases. It can be noted that P2 seems to go to extinction while levels of H2, C1, C2, and P1 significantly decrease with the use of biofuel.

Fig.6 shows the dynamics of the wage rate, price of energy, mass of the energy source, and mass of the human household compartment. During the second half of the simulation, the drop in population is not accompanied by a drop in the mass of the human household compartment. This indicates that in the future, a person will consume more resources (at a higher cost) with and without the use of biofuels. The per capita mass will be lower with the use of biofuels since HH is at a lower level at the end of the simulation.

Fig.6 also shows that the price of energy is the same with or without use of biofuels. The price initially decreases due to the decrease in wages as a result of the increase in population. As the population decreases in the second half of the simulation, the wages increase and therefore the price of energy increases. Like the base case, the deterioration of ES does not occur as quickly with the use of biofuels because some of the energy production is produced from biomass and therefore limits the mass flow of ES to EP.

3.3. Increase in Per Capita Consumption Scenario:

The per capita consumption of mass and resources is continuously increasing. Many of the resources we consume are non-renewable (e.g. fossil fuels) which has led to a fear that this current trend will cause a breakdown in the ecosystem. Although the increase in consumption is difficult to predict, it is believed that the consumption of many resources will increase by

about 50% in the next 50 years.⁶ This is simulated in the model by varying certain coefficients involved in the demand of resources.

The mass fluctuations of some of the ecological compartments for the scenario of an increase in per capita consumption are shown in Fig.7. It can be seen that an increase in consumption levels leads to extinction of P1, P2, H1, H2, C1, and C2. The species reach extinction earlier with the use of biofuels. This is because P1 is also being used for the production of energy and is not available to the other compartments in the ecosystem. Because the model is interconnected, the changes in each compartment affect the entire system.

Fig.8 shows the dynamics of the wage rate, price of energy, mass of the energy source, and the population. It can be seen that there is a drop in population due to the lack of resources such as P1 and H1 which have gone to extinction. This situation shows the catastrophe where limited resources cause loss of human life. The decrease in population occurs sooner with the use of biofuels because compartments reach extinction earlier as seen in Fig.7.

It can also be seen from Fig.8 that an increase in wages occurs sooner with the use of biofuels. The wage rate is inversely proportional to the population and the price of energy is proportional to the wage rate. An interesting observation from this scenario is that the energy source is depleted faster with the use of biofuels. Because P1 reaches extinction by about the 4000th week with the use of biofuels, the energy demand for the rest of the simulation must be met using the non-renewable energy source. The higher price of energy with the use of biofuels indicates that the demand of energy is greater with the use of biofuels than without them. Therefore, more of the energy source is used after 4000 weeks and the energy source is depleted faster using biofuels.

4. Numerical Optimization:

The goal of this work is to extend the lifetime of dying compartments and therefore increase the sustainability of the system, especially for the increase in consumption scenario. Dynamic optimization, specifically non-linear programming (NLP) is applied at a series of time steps where the effect of a regulatory policy on increasing the sustainability of the system is explored.

To solve this optimization problem, a mathematical measure of sustainability is needed as the objective function. Fisher Information (FI), a statistical quantity from information theory, can be interpreted as a measure of stability or order of a system, and can therefore be applied as a measure of sustainability of a system.^{2,9,10} Due to the diverse parameters of the model, FI is a useful measure of stability since it incorporates both the physics and economics of the model.

For a system with n species represented by a set of differential equations, the time average FI is given by equation (1) where T_c is the cycle time and $v(t)$ and $a(t)$ are the velocity and acceleration terms of the ecosystem defined by equations (2) and (3), respectively.⁸

$$I_t = \frac{1}{T_c} \int_0^{T_c} \left(\frac{a(t)^2}{v(t)^4} \right) dt \quad (1)$$

$$v(t) = \sqrt{\sum_{i=1}^n \left(\frac{dx_i}{dt} \right)^2} \quad (2)$$

$$a(t) = \frac{1}{v(t)} \left[\sum_{i=1}^n \frac{dx_i}{dt} \frac{d^2x_i}{dt^2} \right] \quad (3)$$

The terms $\frac{dx_1}{dt}$ through $\frac{dx_n}{dt}$ used in equations (2) and (3) are known as state equations and describe the dynamics of the model in terms of the n state variables. Although these state equations are usually written in differential form as shown in the above equations, the model in this work is defined in terms of difference equations. A functional form of equation (1) therefore needs to be derived for application in this work.

4.1. Problem formulation:

Finite difference formulations for the first and second derivatives are applied to equations (2) and (3) so that the Fisher Information at each time step can be calculated.⁴ The state equations at the first time step are determined using the first order right-sided finite difference scheme defined as

$$\frac{dx}{dt} = \frac{x_{i+1} - x_i}{\Delta x} \quad (4)$$

where $i = 1$ and Δx is the time step length which is one week. The state equations at the final time step are determined using the first order left-sided finite difference scheme defined as

$$\frac{dx}{dt} = \frac{x_i - x_{i-1}}{\Delta x} \quad (5)$$

where $i = 10400$ (final week in the 200 year simulation time). The state equations at all other time steps are determined using the central finite difference scheme defined as

$$\frac{dx}{dt} = \frac{x_{i+1} - x_{i-1}}{2\Delta x} \quad (6)$$

A similar procedure is repeated to determine the finite difference formulations for the second derivatives seen in equation (3).

$$\frac{d^2x}{dt^2} = \frac{x_{i+2} - 2x_{i+1} + x_i}{\Delta x^2} \quad (7)$$

$$\frac{d^2x}{dt^2} = \frac{x_i - 2x_{i-1} + x_{i-2}}{\Delta x^2} \quad (8)$$

$$\frac{d^2x}{dt^2} = \frac{x_{i+1} - 2x_i + x_{i-1}}{\Delta x^2} \quad (9)$$

Equation (7) is used for the initial time step, equation (8) is used for the final time step, and equation (9) is used for all other time steps in the 10400 week time period. These state values can be used in place of the state equations in equations (2) and (3) so that the Fisher Information for each week can be calculated using the following equation

$$I_i = \frac{1}{T} \left(\frac{a(t)^2}{v(t)^4} \right) \quad (10)$$

where I_i is the FI at time step i and T is equal to one week.⁴

Based on this definition of FI, the goal is to develop policies so the system FI is close to the FI of a stable, target system. The target system used in this work is the base case scenario (i.e. no population explosion or increase in per capita consumption) since this is well defined and does not lead to an unstable system. The objective function for the minimization of FI variance is defined as

$$J = \text{Min} \int_0^T (I(t) - I_c(t))^2 dt \quad (11)$$

where T is the total time under consideration, $I(t)$ is the current FI profile, and $I_c(t)$ is the targeted FI profile (base case). The functional form for a discretized model is defined as

$$J = \text{Min} \sum_{i=1}^N (I(i) - I_c(i))^2 \quad (12)$$

where N is the total number of model cycles, $I(i)$ is the average FI for the i^{th} model cycle for the current profile and $I_c(i)$ is the average FI for the i^{th} model cycle for the base case scenario.⁸

The model in this work is discretized into time steps of 5 years for a total number of 40 cycles for the 200 year time period. Therefore, $I(i)$ is the average FI every five years for the current model and $I_c(i)$ is the average FI every five years for the base case scenario. This discretization of the simulation time is described at the end of this section.

The dynamic optimization can be performed in Matlab according to the strategy outlined in Fig.9. Optimization using non-linear programming (NLP) is used in this work due to the non-

linearity of the objective function. The optimization is initialized with a starting value for the decision variable. The objective function is calculated by trying to satisfy optimality conditions known as Karush-Kuhn-Tucker conditions (KKT). For an unconstrained NLP the optimality condition is defined as

$$\nabla J(x) = 0 \quad (13)$$

where J is the objective function and x is the decision variable. The optimizer calculates a new value for the decision variable and this iterative sequence continues until the optimization criteria (KKT) are met. A policy is used as the decision variable at each time step where the value is determined by minimizing the objective function defined in equation (6). A profile of the policy variable for the simulation time can then be created.³

Three possible policy variables are investigated in this work:⁸

1. Governmental policy variable: Discharge fee charged to the industrial sector (p_{ISHH})
2. Policy related to the efficiency of technology: Amount of P1 required to produce a unit of the IS product (θ)
3. Governmental policy variable: Amount of P2 consumed by H1 through grazing (\hat{k})

Fig.10 shows an example of the discretization of the 200 year time period where θ is the policy variable. For each five year time step, the value of θ which minimizes the objective function is determined using the discretized optimization formulation described above. The average FI for each five year time interval is obtained by averaging the 260 week FI values within the interval calculated from equation (10). The average FI every five years is determined for the current profile and base case to be used in equation (12).

5. Techno-economic Policy Results:

5.1. Discharge Fee as a policy variable:

The optimization problem is first solved by using the discharge fee charged to the industrial sector as the policy (control) variable. The base value for the discharge fee is 1×10^{-8} and the lower and upper bounds for the control variable are 1×10^{-9} and 1×10^{-7} , respectively. The profile of the policy variable with and without the use of bioenergy is shown in Fig.11. The average increase in the policy variable is about 9.5 times the base case value both with and without the use of bioenergy. The dynamics of P1, H1, C1, and the human population without bioenergy are shown in Fig.12. There is a significant increase in the lifetime of all parameters because P1 no longer reaches extinction in the simulation time. This is because the increase in the discharge fee causes the demand of IS to decrease. Consequently, the demand of P1 by IS significantly decreases which therefore extends the lifetime of P1.⁸

The dynamics of the same compartments with the use of bioenergy are shown in Fig.13. P1 reaches extinction at about the 7000th week due to the fact that some P1 is being used to produce energy, extending the lifetime of P1 by about 60 years. H1 and the human population begin to decrease towards the end of the simulation due to the extinction of P1.

Hence, the use of the discharge fee as a policy variable is effective in extending the lifetime of dying compartments.

5.2. θ as a policy variable:

The amount of P1 required to produce a unit of the IS product (θ) is then used as the policy variable. The base value for θ is 0.102 and the lower and upper bounds for the control variable are 0.001 and 1.0, respectively. The profile of the policy variable with and without the use of bioenergy is shown in Fig.14. There is an average decrease in θ which results in marginal improvement in the model stability. The dynamics of P1, H1, C1, and the human population without bioenergy are shown in Fig.15. The lifetime of all parameters slightly increase due to the delayed extinction of P1 of almost three years. The decline of the human population is also delayed when θ is controlled.

The dynamics of the same compartments with the use of bioenergy are shown in Fig.16. Similarly, there is a slight improvement in the lifetime of the dying compartments. The extinction of P1 is delayed by about 2.5 years which results in similar improvements in the model dynamics for other ecological compartments. The decline of the human population is also marginally delayed.

Although the use of θ as a policy variable does increase the lifetime of dying compartments, the improvement is insignificant for practical purposes and θ is therefore not an effective policy variable.

5.3. \hat{k} as a policy variable:

The amount of P2 consumed by H1 through grazing (\hat{k}) is the final parameter used as a policy variable. The base value for \hat{k} is 0.09 and the lower and upper bounds are 0 and 1.0, respectively. The profile of the policy variable with and without the use of bioenergy is shown in Fig.17. The dynamics of H2 and C1 with and without bioenergy are shown in Fig.18 and Fig.19, respectively. It is observed that the lifetime of H2 and C1 is not significantly impacted by \hat{k} . This policy affects P2 more than H1, which means any changes to the system are not seen in the domesticated sector. In addition, H1, H2 and H3 all consume P2, so any change in the consumption by H1 is simply met by one of the other herbivore compartments.

Although fluctuations in H2 and C1 can be observed in both Fig. 18 and Fig.19, these do not affect any major changes in the system stability. \hat{k} is therefore not an effect policy variable.

6. Discussion and Summary:

From the two scenario simulations we can conclude that sustainability of even a simple ecosystem may not be necessarily intuitive. Especially for the increase in per capita consumption scenario, the use of biomass as a source of energy accelerates the extinction of species and the depletion of the non-renewable energy source which may not have been a foreseen outcome. The scenario studies suggest that an increase in per capita consumption is more critical than a population explosion and future regulatory strategies should focus on the reduction of consumption rates.

Numerical optimization is used to simulate time dependent regulatory strategies by controlling three possible parameters: the discharge fee charged to the industrial sector, mass of P1 necessary to produce one unit of the IS product, and the amount of P2 consumed by H1. The results show that controlling the discharge fee charged to the IS is most effective in prolonging the lifetime of dying compartments in the model. This however, is not a long term solution and other policies should be explored.

It should be noted that this data can only be taken in a general sense due to the simplicity of the model when compared to the planet. Although no specific system is used in this study, the hope is that this model, perhaps with further enhancement, could be used to simulate the consequences of actual environmental management strategies. More applicable results will depend on the success of models to replicate reality.

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Figure 1: Integrated ecological-economic compartmental model of a simplified planet. Arrows represent transfer of mass between compartments.

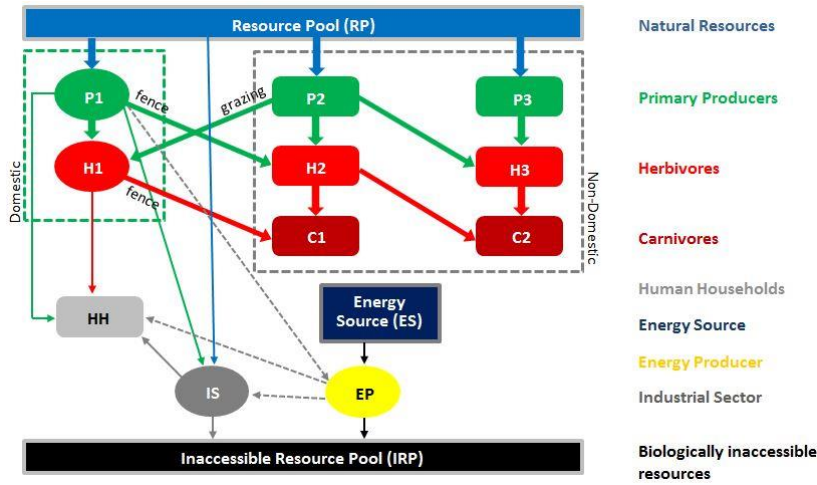


Figure 2: Profiles of primary producer 1, herbivore 1, and carnivore 1 (base case).

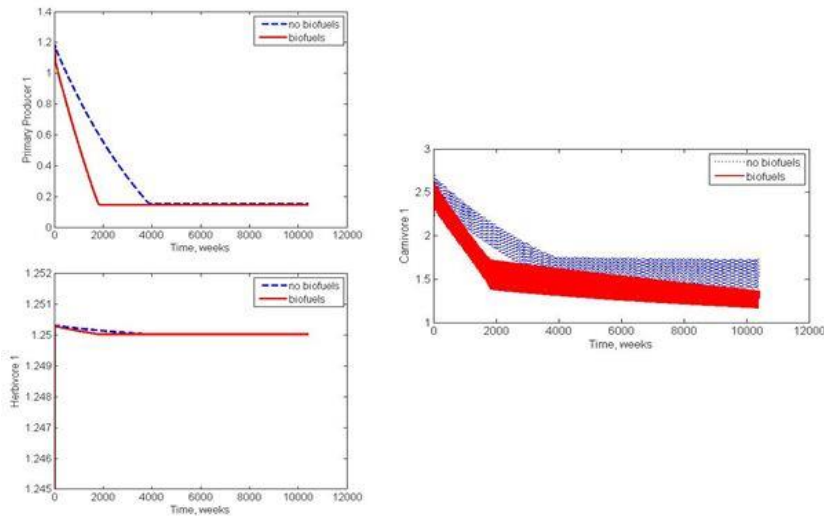


Figure 3: Profiles of human population, mass of human households, and mass of the energy source (base case).

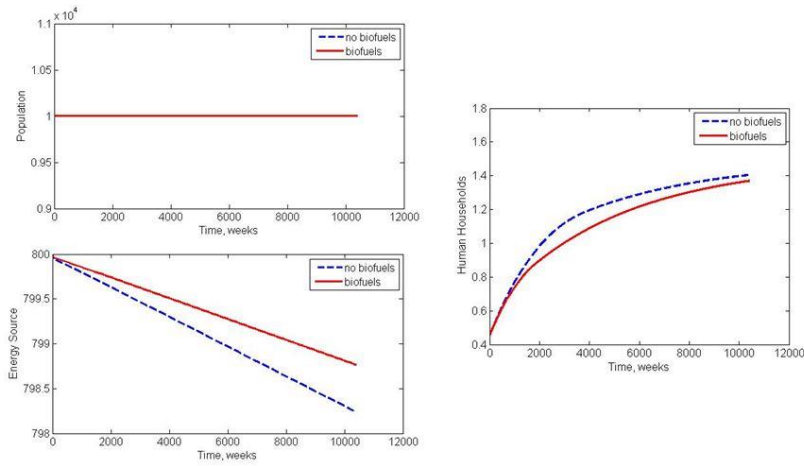


Figure 4: Profile of human population (population explosion).

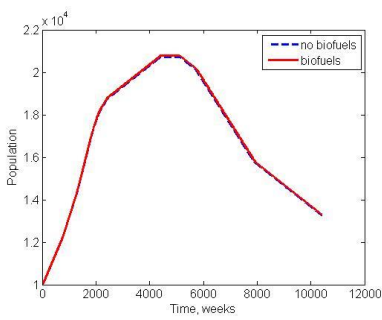


Figure 5: Profiles of ecological compartments P1, P2, H1, H2, C1, and C2 (population explosion).

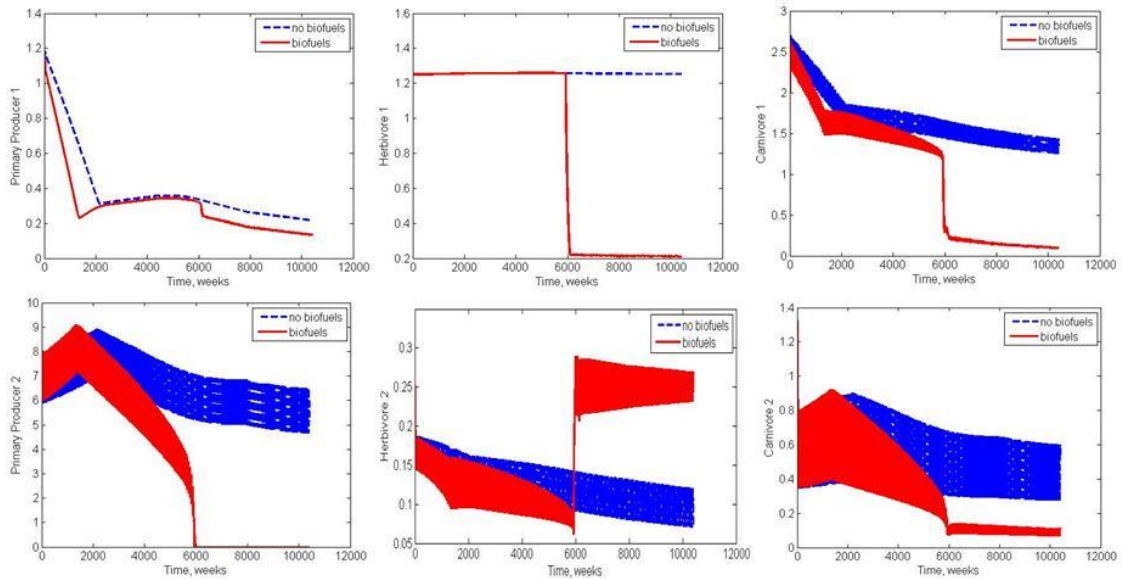


Figure 6: Profiles of wages, price of energy, mass of the energy source, and mass of the human households (population explosion).

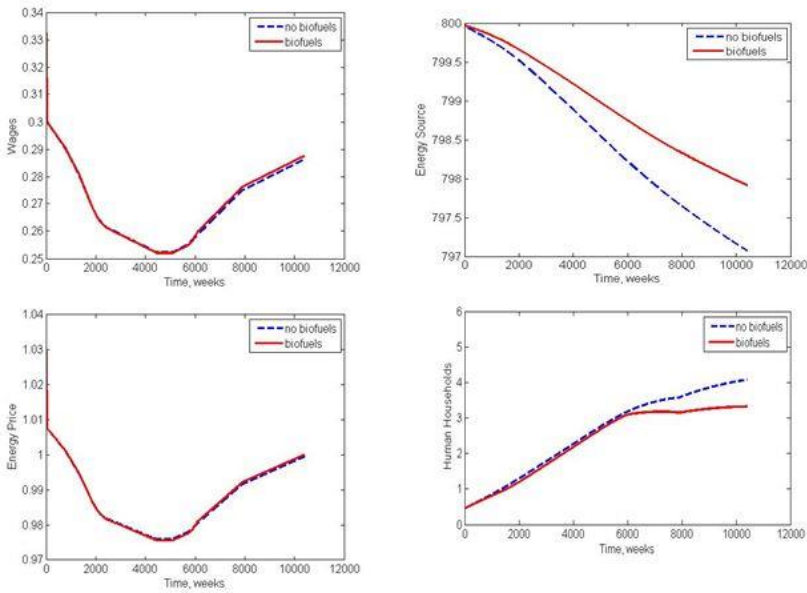


Figure 7: Profiles of ecological compartments P1, P2, H1, H2, C1, and C2 (increase in per capita consumption).

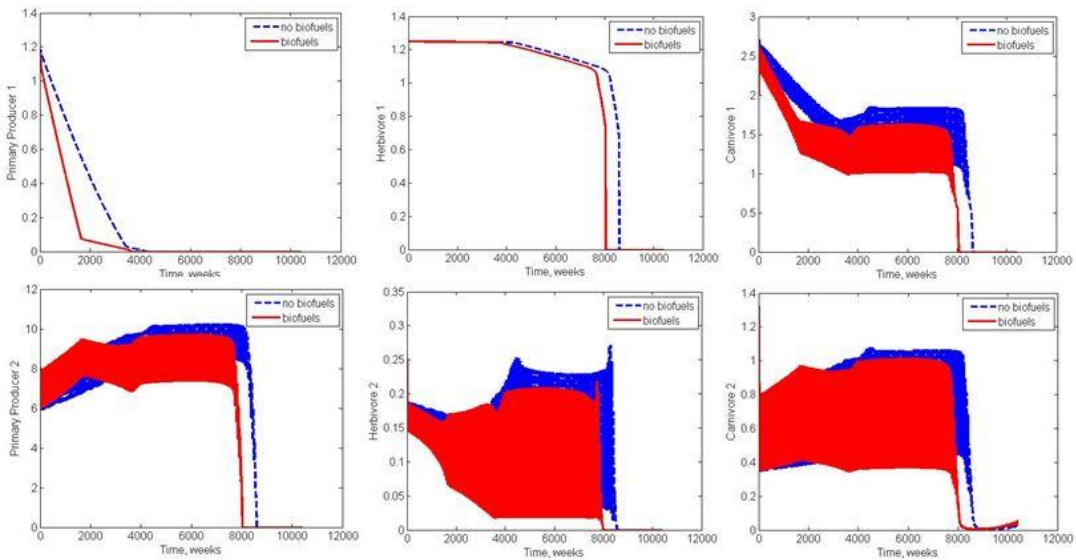


Figure 8: Profiles of wages, price of energy, mass of the energy source, and human population (increase in per capita consumption).

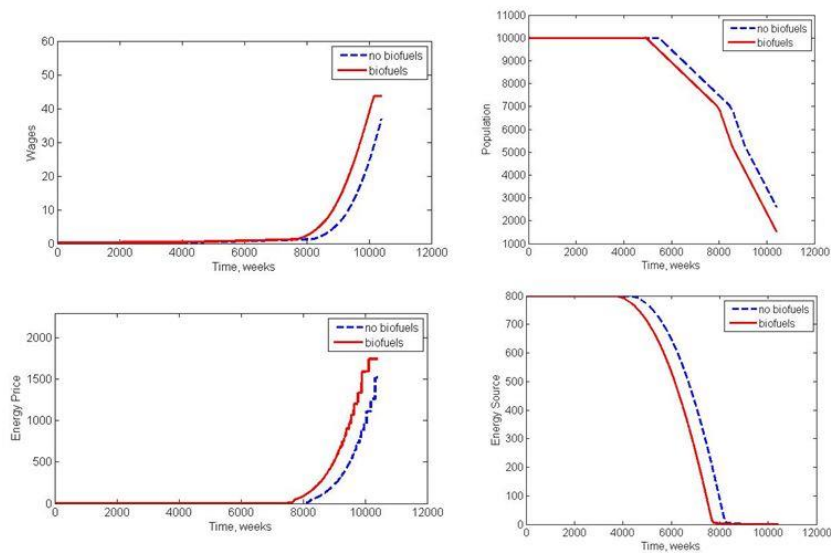


Figure 9: Pictorial representation of the iterative procedure for NLP optimization.

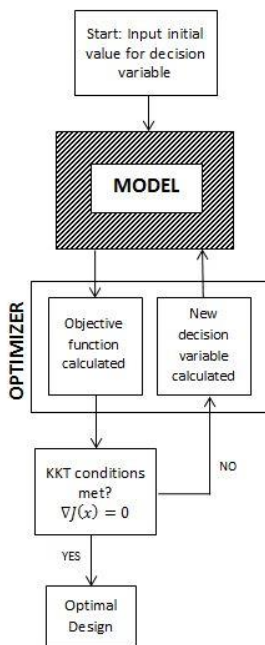


Figure 10: Value of the policy variable θ which optimizes the objective function at each discretized time step (no bioenergy).

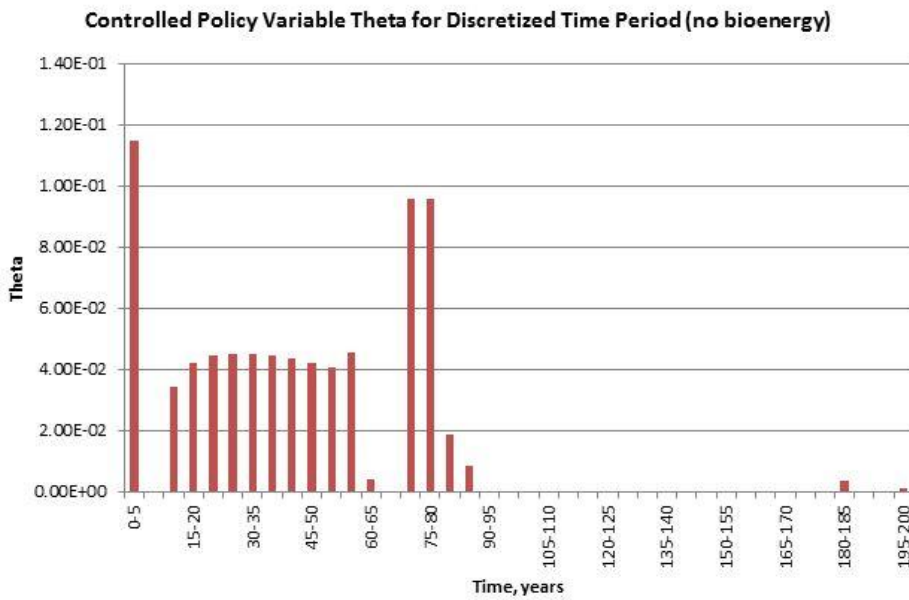


Figure 11: Discharge fee profile for the uncontrolled and controlled model (with and without bioenergy) for the increase in per capita consumption

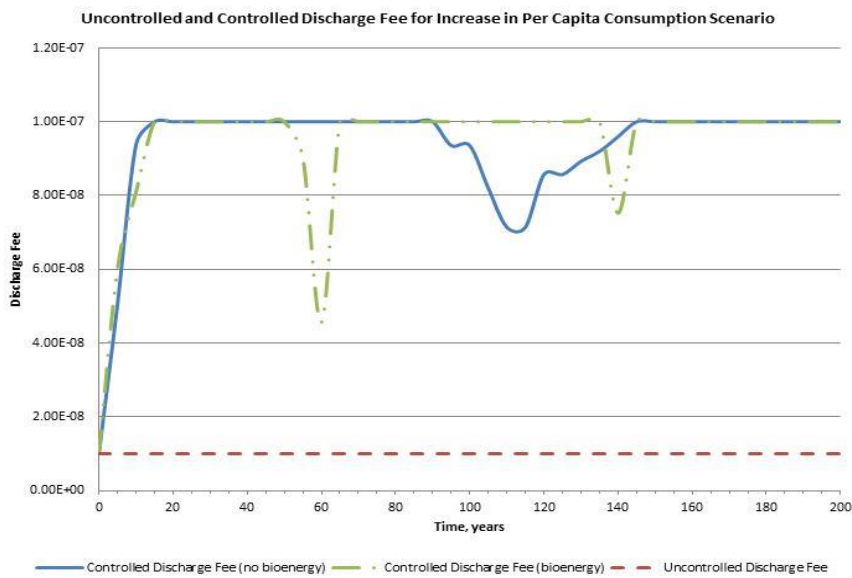


Figure 12: Discharge Fee controlled: Profiles of primary producer 1, herbivore 1, carnivore 1, and human population without bioenergy (increase in per capita consumption).

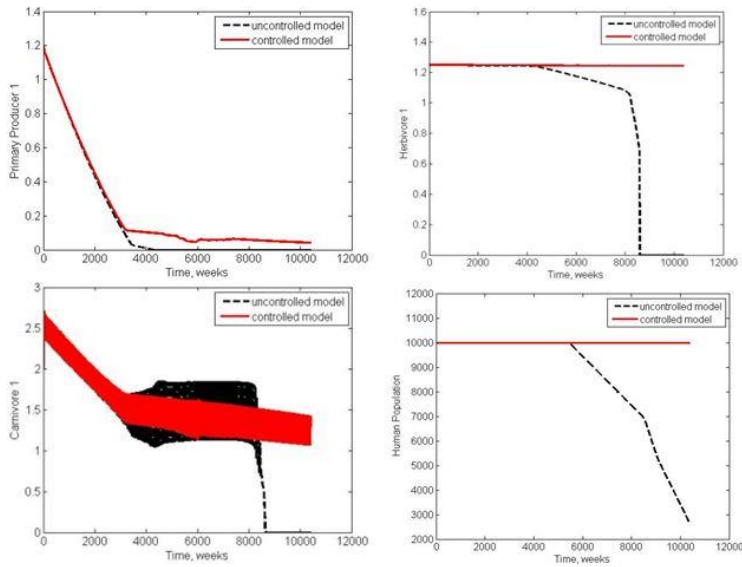


Figure 13: Discharge Fee controlled: Profiles of primary producer 1, herbivore 1, carnivore 1, and human population with bioenergy (increase in per capita consumption).

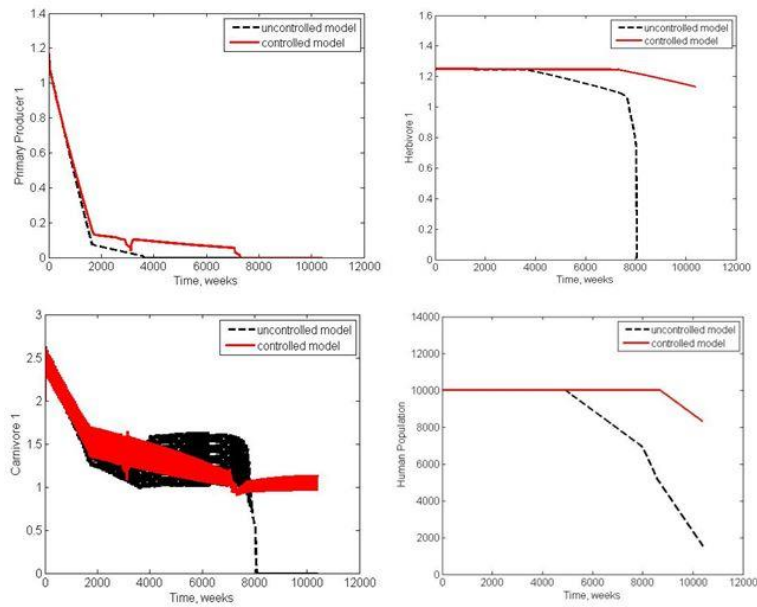


Figure 14: θ profile for the uncontrolled and controlled model (with and without bioenergy) for the increase in per capita consumption scenario.

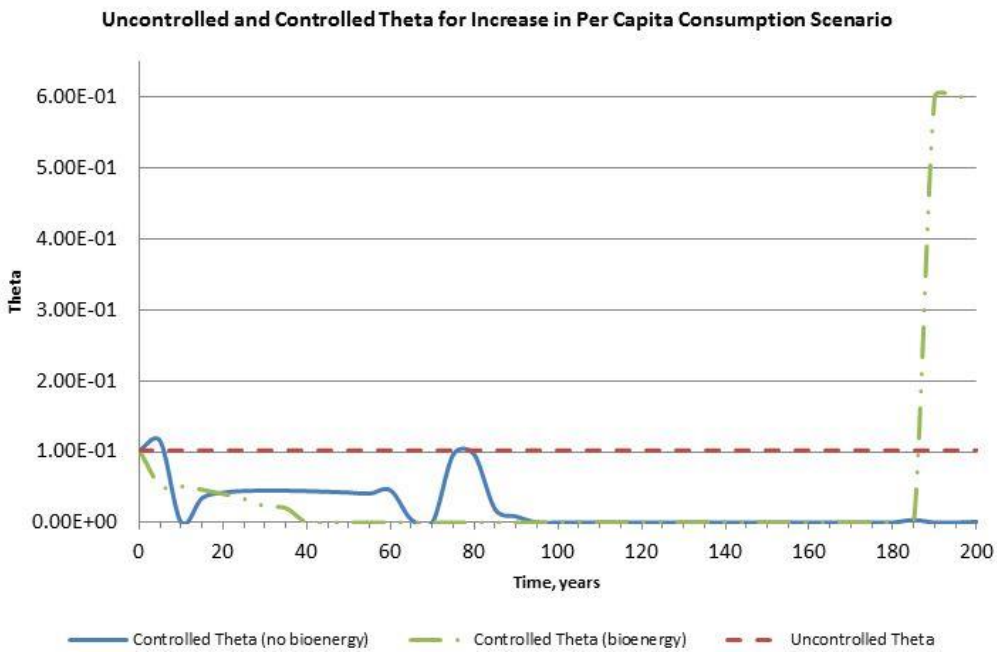


Figure 15: θ controlled: Profiles of primary producer 1, herbivore 1, carnivore 1, and human population without bioenergy (increase in per capita consumption).

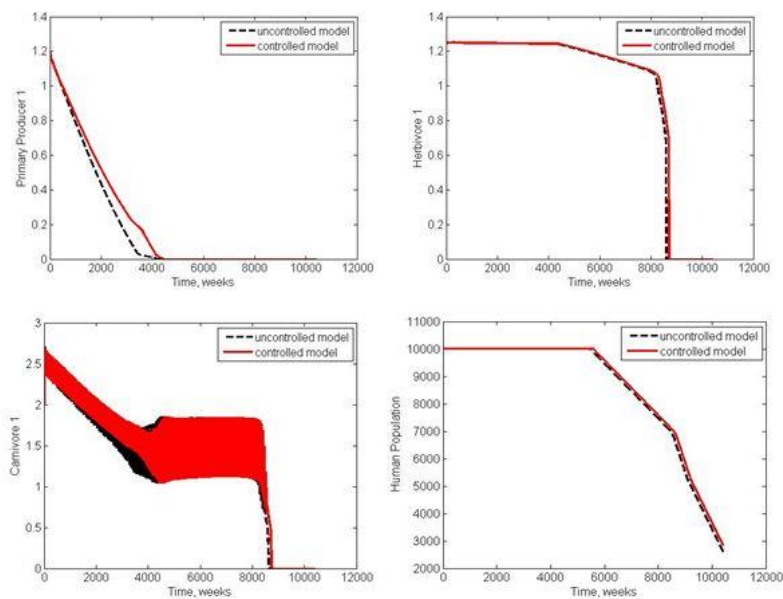


Figure 16: θ controlled: Profiles of primary producer 1, herbivore 1, carnivore 1, and human population with bioenergy (increase in per capita consumption).

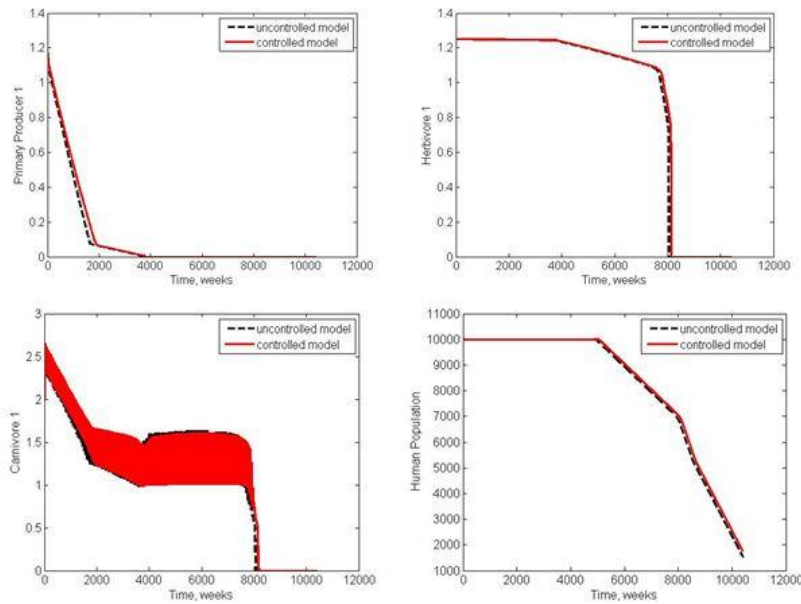


Figure 17: \hat{k} profile for the uncontrolled and controlled model (with and without bioenergy) for the increase in per capita consumption scenario.

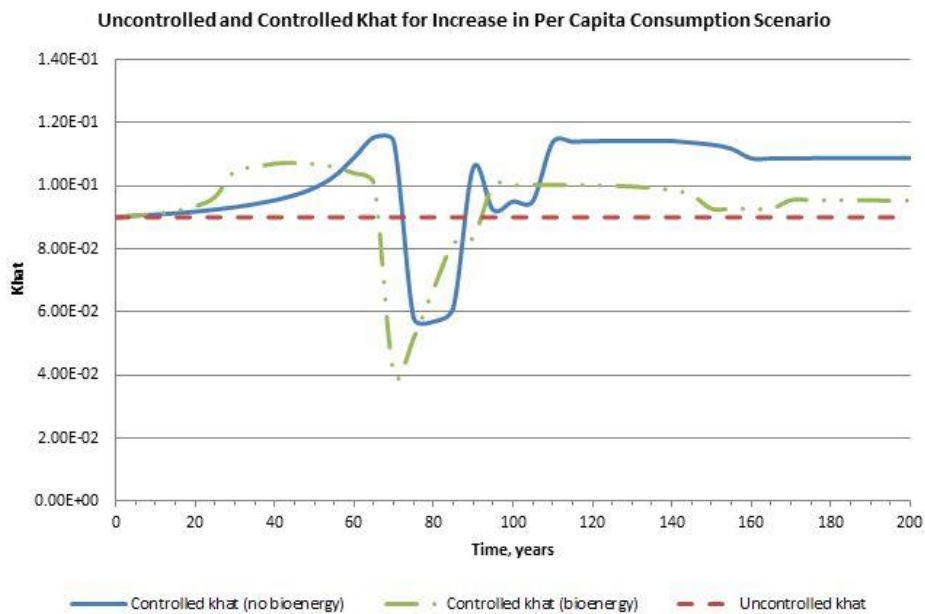


Figure 18: \hat{k} controlled: Profiles of carnivore 1 and herbivore 2 without bioenergy (increase in per capita consumption).

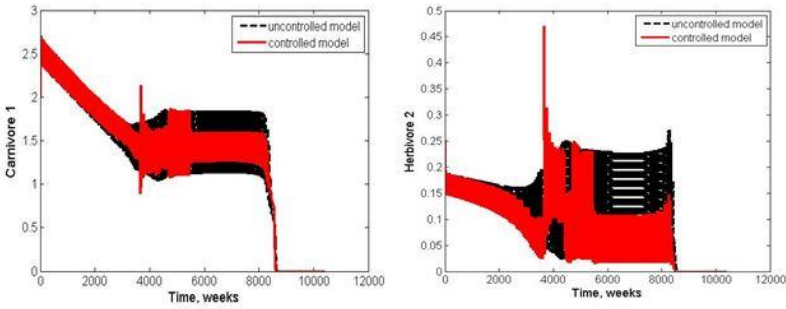


Figure 19: \hat{k} controlled: Profiles of carnivore 1 and herbivore 2 with bioenergy (increase in per capita consumption).

