Intracranial Blood Pressure and Brain Vasculature

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Motivation

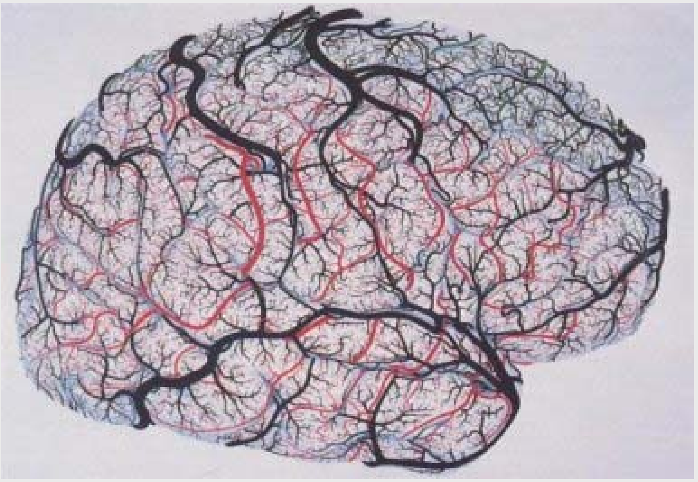
Brain vasculature:

Arrangement of blood vessels in the brain

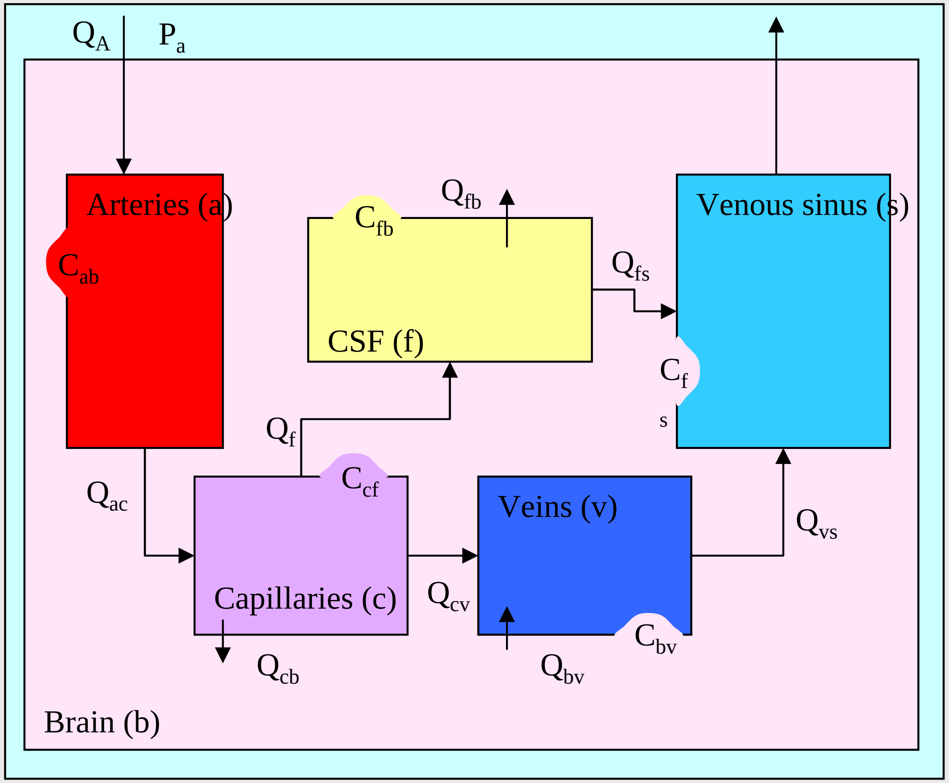
Why study brain vasculature?

- **More than 80 million people in the world are affected by neurodegenerative diseases of the central nervous system (NIH, 2005)**
- **Current models fail to establish a relationship between blood pressure and blood flow rates for such pathological conditions**
- **Intracranial dynamics**
  - Interaction of blood, CSF, and soft brain tissue
  - A quantitative understanding is required to improve treatment and diagnosis
- **Flow physics of the brain is not understood**

Previous Research: Compartmental Model



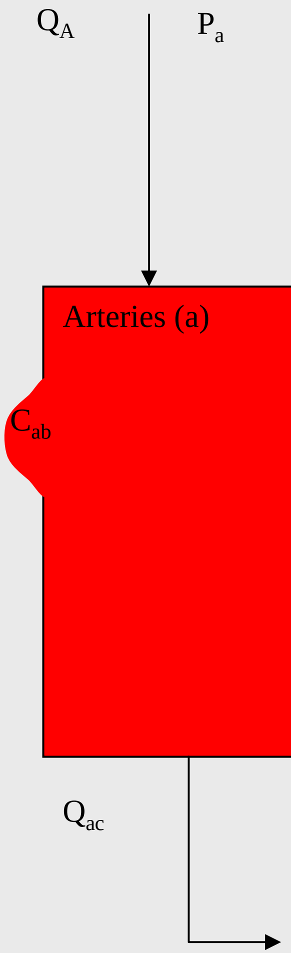
Brain vasculature



Compartmental model:

- **Blood flow**
- **Distensibility**
- **Blood Pressure**

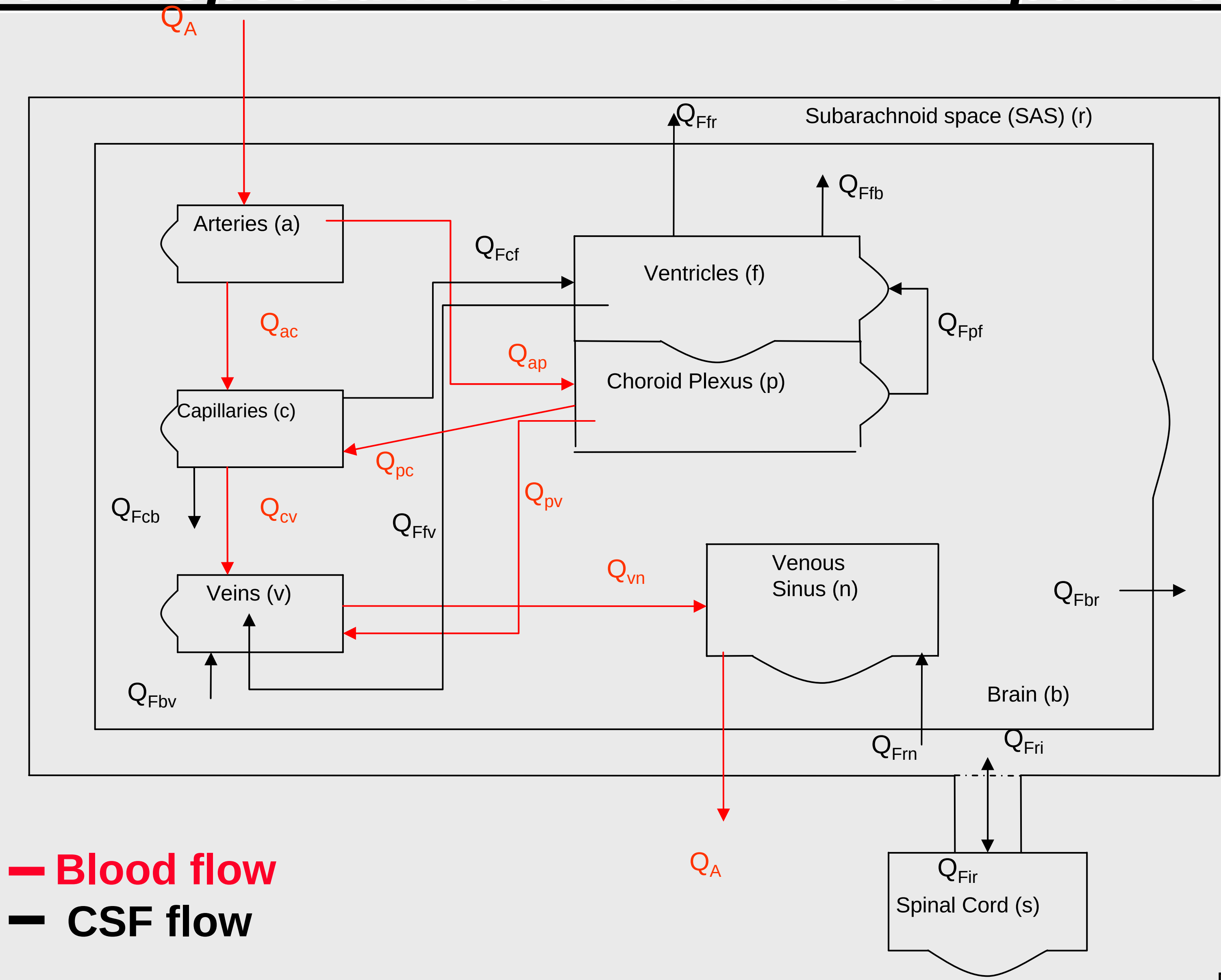
Physiological Inconsistencies



- **Cranial volume decreases**
  - Physiologically inconsistent
  - Solid structure
- **No distinction between blood flow and CSF flow**
- **Spinal cord unaccounted for**
  - Required to model the pulsatile CSF displacement
- **Steady State**

Solution: Physiologically consistent network model

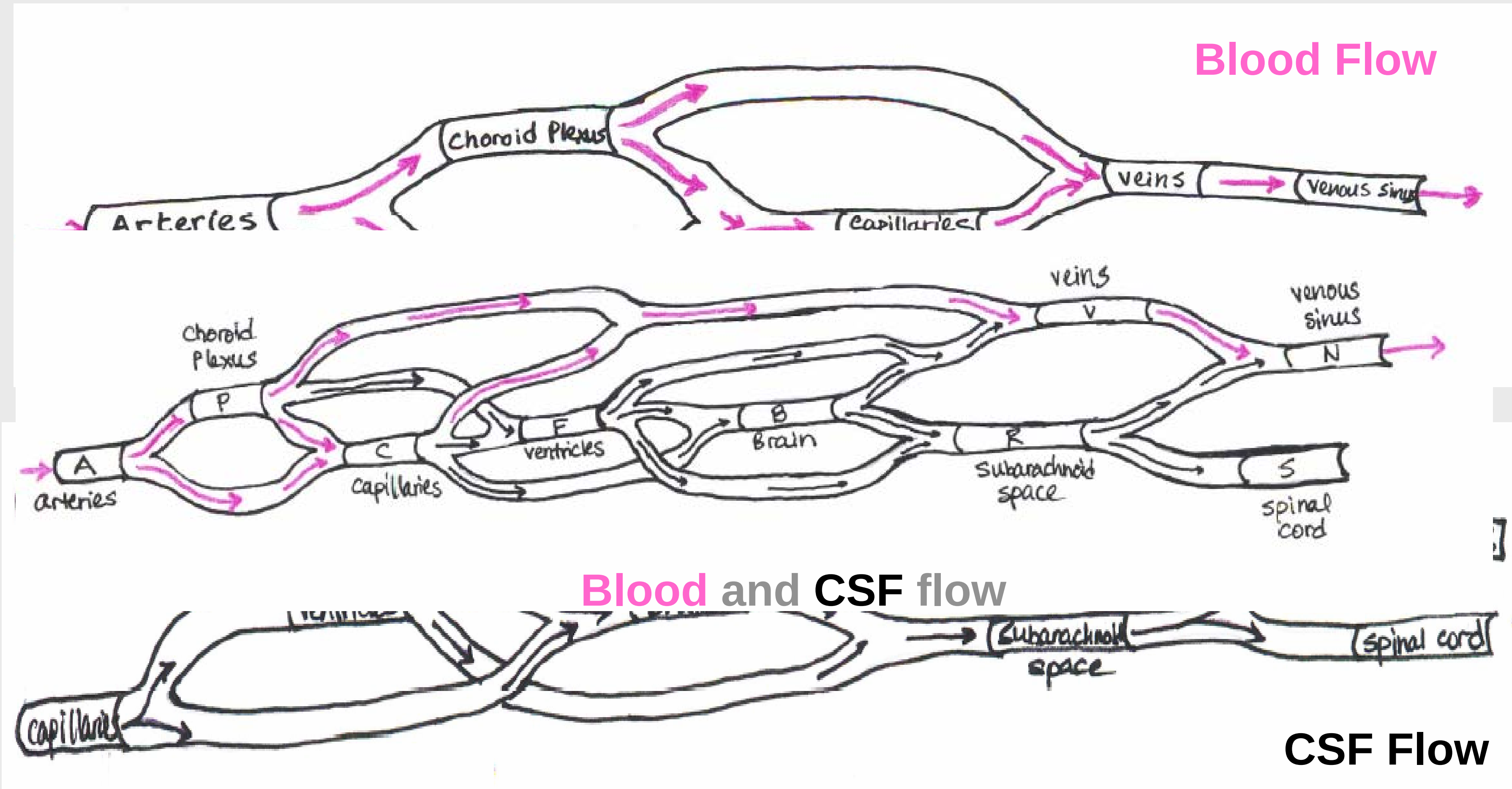
New Proposed Model with nine Compartments



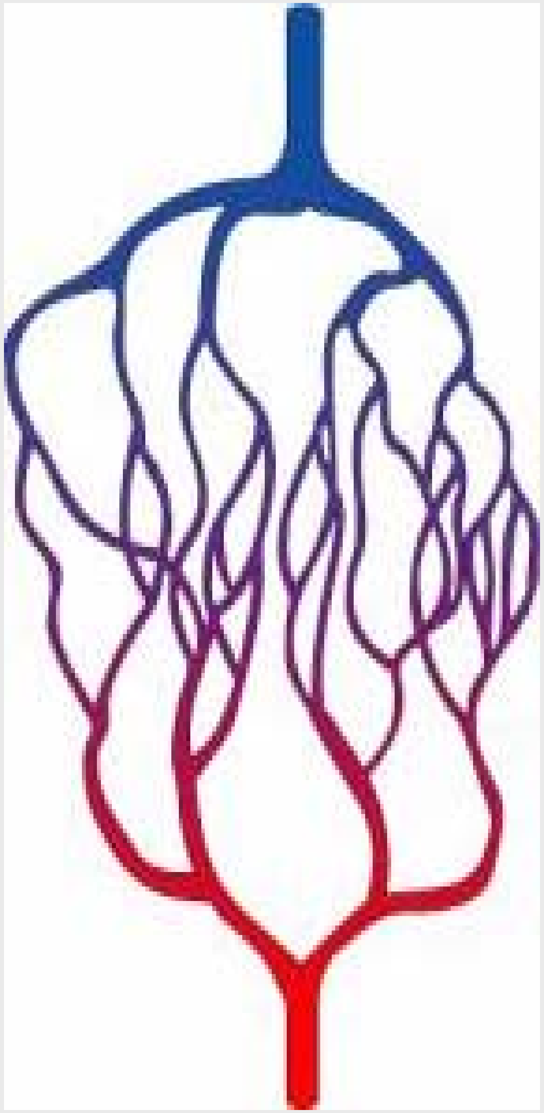
Blood flow

CSF flow

Network of Distensible Tubes



Balance Equations



Continuity:

$$\frac{\partial A}{\partial t} + \frac{\partial (AU)}{\partial x} = 0 \qquad (1)$$

Momentum balance:

$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial x}\left(\frac{U^2}{2} + \frac{P}{\rho}\right) = -F \qquad (2)$$

Extensibility of elastic blood vessels (Tube Law)

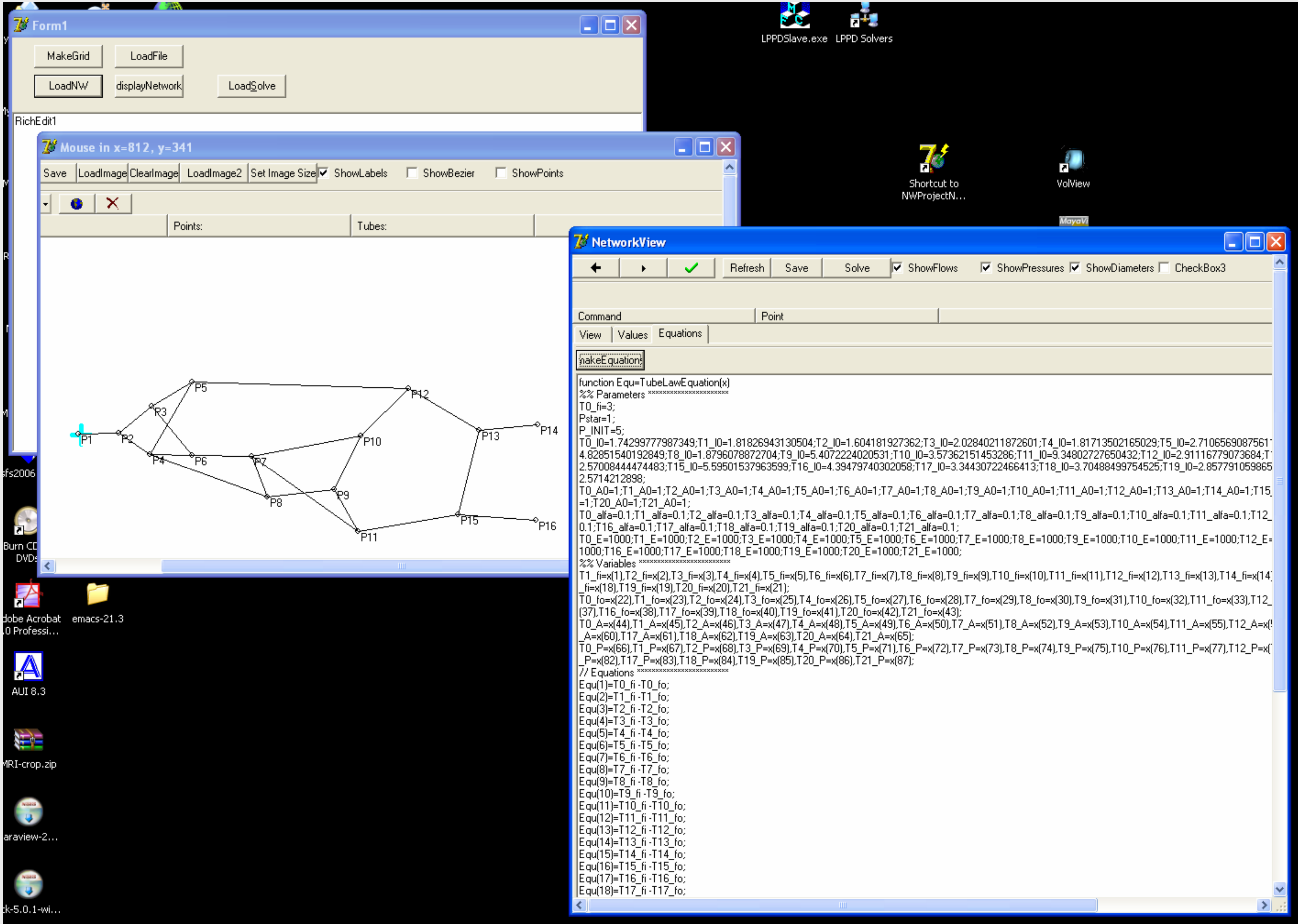
$$P = E_L\left(\frac{A}{A_o} - 1\right) \qquad (3)$$

Methods

- **Use continuity, momentum balance, and the distensibilty equations**

- **Model generator**
  - Developed by Professor Linninger
  - Simple network
  - Equations

- **Equations implemented in a MATLAB program**
  - Steady state and dynamic results obtained
  - A total of 21 tubes
  - 87 unknowns, 87 equations



Form1
MakeGrid
LoadFile
LoadNW
displayNetwork
LoadSolve
RichEdit1
LPPDSlave.exe
LPPD Solvers
Mouse in x=812, y=341
Save
LoadImage
ClearImage
LoadImage2
Set Image Size
ShowLabels
ShowBezier
ShowPoints
Points:
Tubes:
Shortcut to NWProjectN...
VolView
P1
P2
P3
P4
P5
P6
P7
P8
P9
P10
P11
P12
P13
P14
P15
P16
NetworkView
Refresh
Save
Solve
ShowFlows
ShowPressures
ShowDiameters
CheckBox3
Command
Point
View
Values
Equations
nakeEquations
function Equ=TubeLawEquation(x)
%% Parameters ************************
T0_fi=3;
Pstar=1;
P_INIT=5;
T0_l0=1.74299777987349;T1_l0=1.81826943130504;T2_l0=1.604181927362;T3_l0=2.02840211872601;T4_l0=1.81713502165029;T5_l0=2.7106569087561
4.82851540192849;T8_l0=1.8796078872704;T9_l0=5.4072224020531;T10_l0=3.57362151453286;T11_l0=9.34802727650432;T12_l0=2.91116779073684;T
2.57008444474483;T15_l0=5.59501537963599;T16_l0=4.39479740302058;T17_l0=3.34430722466413;T18_l0=3.70488499754525;T19_l0=2.857791059865
2.5714212898;
T0_A0=1;T1_A0=1;T2_A0=1;T3_A0=1;T4_A0=1;T5_A0=1;T6_A0=1;T7_A0=1;T8_A0=1;T9_A0=1;T10_A0=1;T11_A0=1;T12_A0=1;T13_A0=1;T14_A0=1;T15_
=1;T20_A0=1;T21_A0=1;
T0_alfa=0.1;T1_alfa=0.1;T2_alfa=0.1;T3_alfa=0.1;T4_alfa=0.1;T5_alfa=0.1;T6_alfa=0.1;T7_alfa=0.1;T8_alfa=0.1;T9_alfa=0.1;T10_alfa=0.1;T11_alfa=0.1;T12_
0.1;T16_alfa=0.1;T17_alfa=0.1;T18_alfa=0.1;T19_alfa=0.1;T20_alfa=0.1;T21_alfa=0.1;
T0_E=1000;T1_E=1000;T2_E=1000;T3_E=1000;T4_E=1000;T5_E=1000;T6_E=1000;T7_E=1000;T8_E=1000;T9_E=1000;T10_E=1000;T11_E=1000;T12_E=
1000;T16_E=1000;T17_E=1000;T18_E=1000;T19_E=1000;T20_E=1000;T21_E=1000;
%% Variables **************************
T1_fi=x(1),T2_fi=x(2),T3_fi=x(3),T4_fi=x(4),T5_fi=x(5),T6_fi=x(6),T7_fi=x(7),T8_fi=x(8),T9_fi=x(9),T10_fi=x(10),T11_fi=x(11),T12_fi=x(12),T13_fi=x(13),T14_fi=x(14)
_fi=x(18),T19_fi=x(19),T20_fi=x(20),T21_fi=x(21);
T0_fo=x(22),T1_fo=x(23),T2_fo=x(24),T3_fo=x(25),T4_fo=x(26),T5_fo=x(27),T6_fo=x(28),T7_fo=x(29),T8_fo=x(30),T9_fo=x(31),T10_fo=x(32),T11_fo=x(33),T12_
(37),T16_fo=x(38),T17_fo=x(39),T18_fo=x(40),T19_fo=x(41),T20_fo=x(42),T21_fo=x(43);
T0_A=x(44),T1_A=x(45),T2_A=x(46),T3_A=x(47),T4_A=x(48),T5_A=x(49),T6_A=x(50),T7_A=x(51),T8_A=x(52),T9_A=x(53),T10_A=x(54),T11_A=x(55),T12_A=x(
_A=x(60),T17_A=x(61),T18_A=x(62),T19_A=x(63),T20_A=x(64),T21_A=x(65);
T0_P=x(66),T1_P=x(67),T2_P=x(68),T3_P=x(69),T4_P=x(70),T5_P=x(71),T6_P=x(72),T7_P=x(73),T8_P=x(74),T9_P=x(75),T10_P=x(76),T11_P=x(77),T12_P=x(
_P=x(82),T17_P=x(83),T18_P=x(84),T19_P=x(85),T20_P=x(86),T21_P=x(87);
// Equations **************************
Equ(1)=T0_fi -T0_fo;
Equ(2)=T1_fi -T1_fo;
Equ(3)=T2_fi -T2_fo;
Equ(4)=T3_fi -T3_fo;
Equ(5)=T4_fi -T4_fo;
Equ(6)=T5_fi -T5_fo;
Equ(7)=T6_fi -T6_fo;
Equ(8)=T7_fi -T7_fo;
Equ(9)=T8_fi -T8_fo;
Equ(10)=T9_fi -T9_fo;
Equ(11)=T10_fi -T10_fo;
Equ(12)=T11_fi -T11_fo;
Equ(13)=T12_fi -T12_fo;
Equ(14)=T13_fi -T13_fo;
Equ(15)=T14_fi -T14_fo;
Equ(16)=T15_fi -T15_fo;
Equ(17)=T16_fi -T16_fo;
Equ(18)=T17_fi -T17_fo;
AUI 8.3
emacs-21.3

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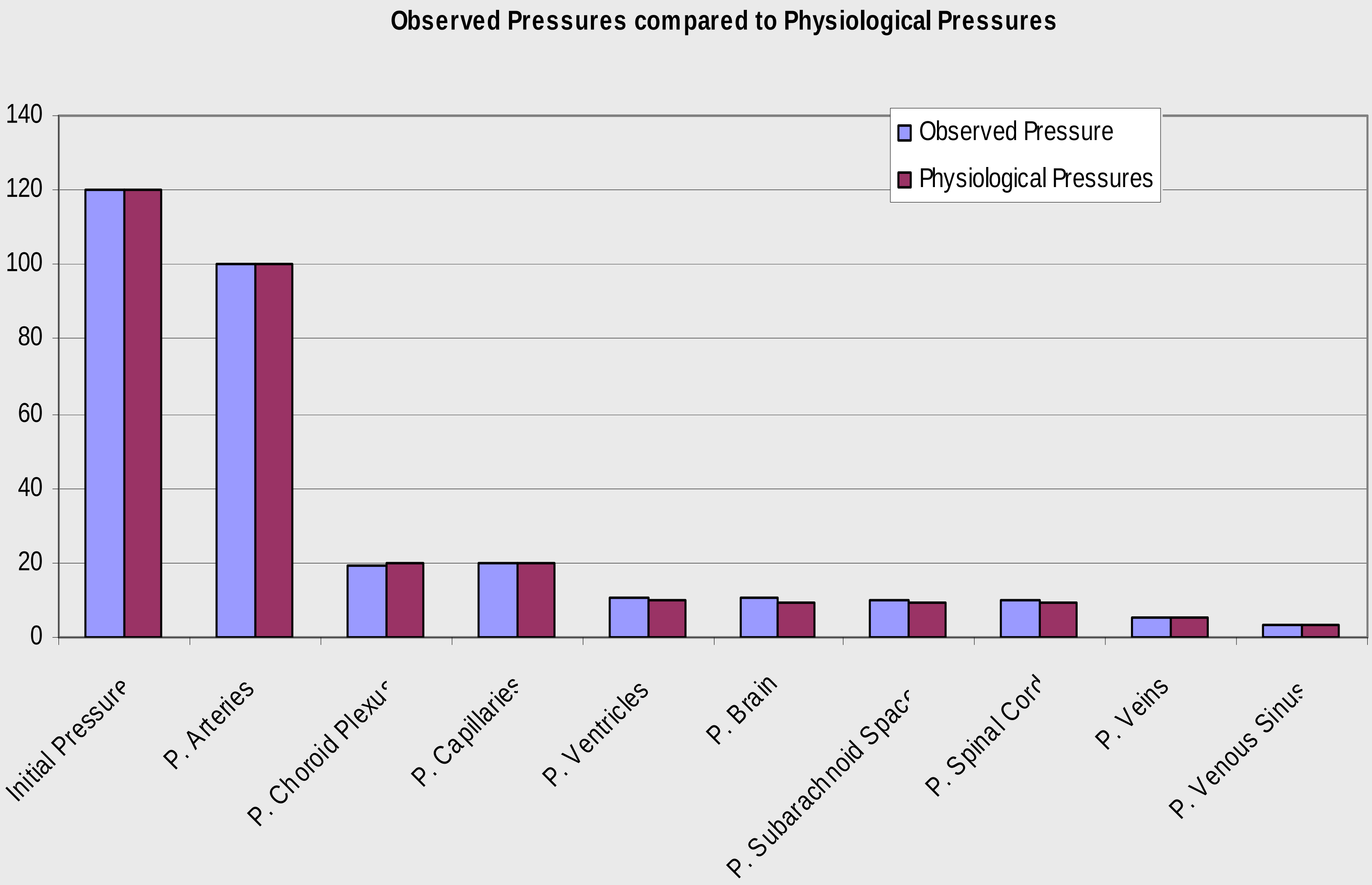
Pressure Drops



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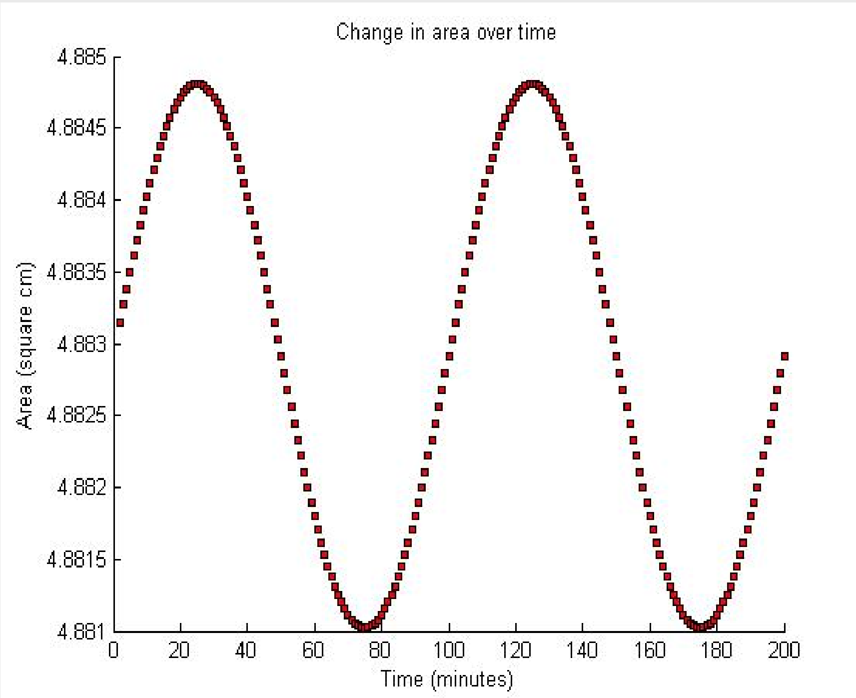
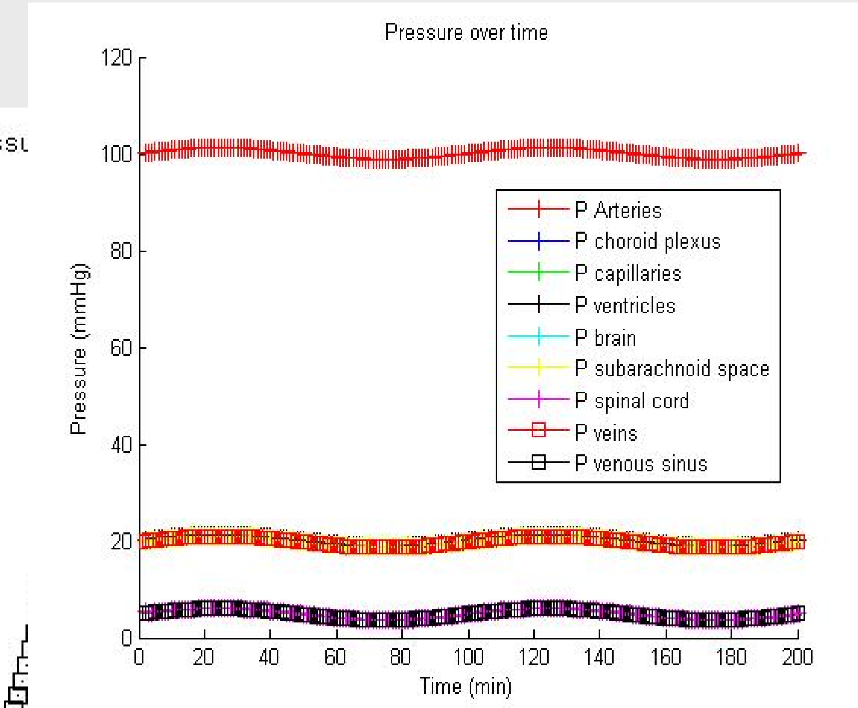
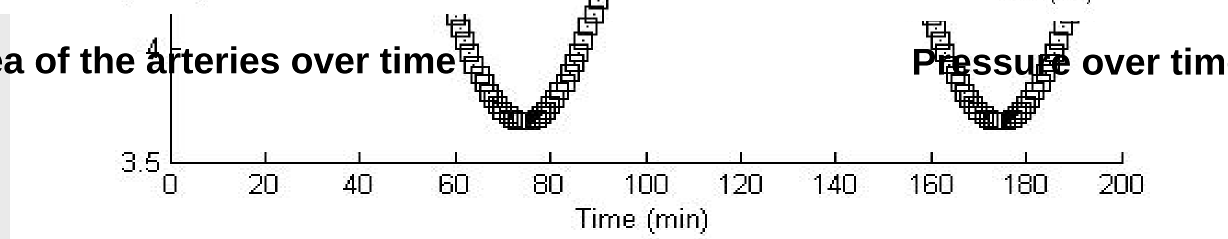
Entire Network Results

Spinal Cord Pressure Change over time







Change in area of the arteries over time

Pressure over time

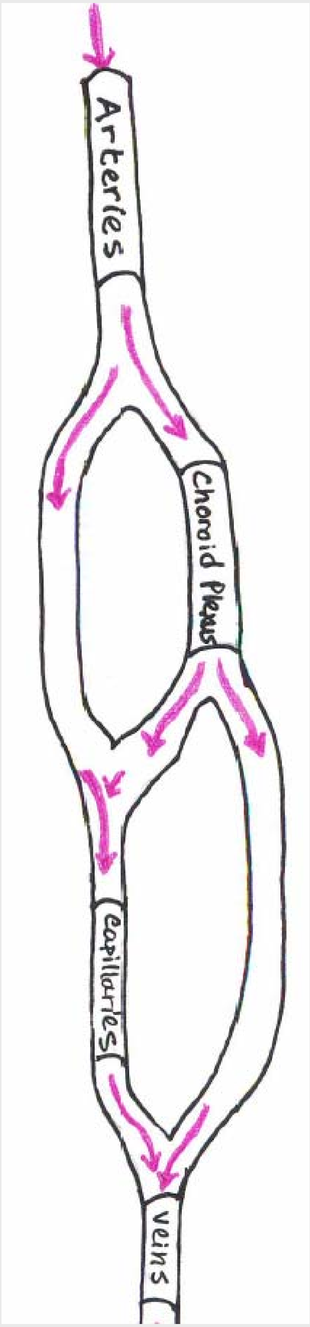
Conclusions/ Future Directions

- **Network is physiologically consistent**
- **Network generator used to produce equations accurately for numerous tubes**
- **Model various conditions**
  - Hydrocephalus
  - High blood pressure

- **Superimpose brain vasculature on the solid brain**
  - Study the effects of brain injury or trauma
    - » Effect on vasculature and the surrounding brain tissue
  - Brain deformation affects vasculature and the tissue

- **Study impacts of a stroke, high blood pressure**

- **Understand the effects of tumor growth on the compressed vasculature and tissue**



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- **Dr. Libin Zhang**
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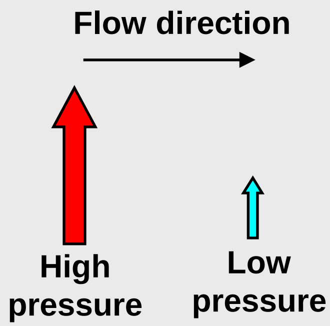
Equations and Balances

Pressure driven flows:

Impulse balance

$$Q_{ij} = \frac{P_i - P_j}{R_{ij}}$$



Closed Cranium:

$$Q_A = Q_J$$

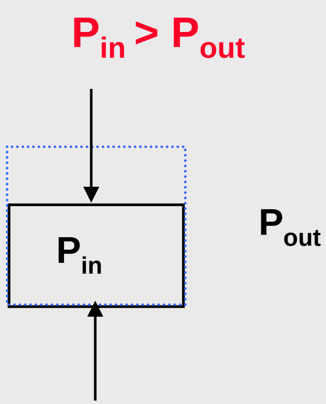
- Cranial volume is considered constant
- Any input must be compensated by an equal output

Distensibility of the compartment:

Deformation of the membrane between adjacent compartments.

$$\Delta V = C * \Delta P$$

$$\frac{dV_{ij}}{dt} = C_{ij} \frac{d(P_i - P_j)}{dt}$$



C_{ij} denotes the compliance between the two components

Compliance elements indicate that an increase in volume of one compartment equals the volume of the cup formed by the deformed membrane (Karni et al, 1988)

Arteries (blood flow)

(1) Continuity:

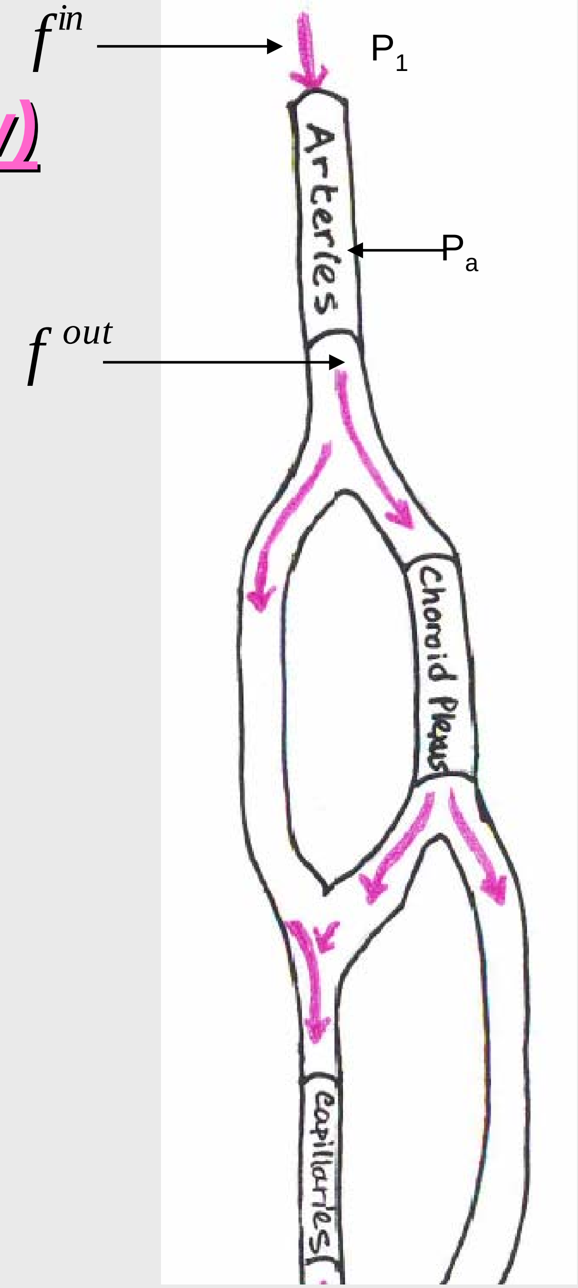
$$\frac{dA_a}{dt} = f^{in} - f^{out}$$

(2) Momentum:

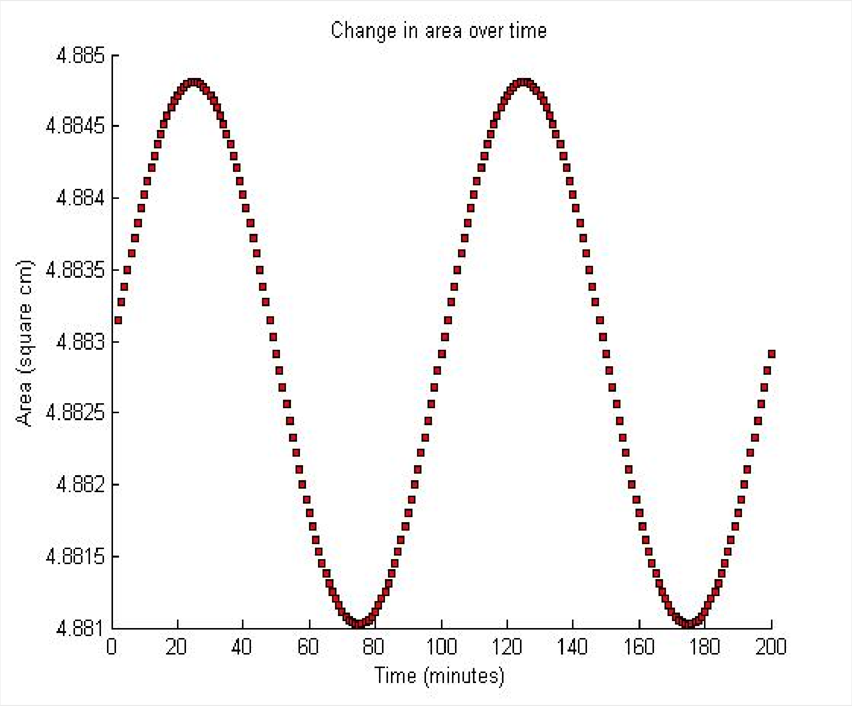
$$P_1 - P_a = \alpha f^{in}$$

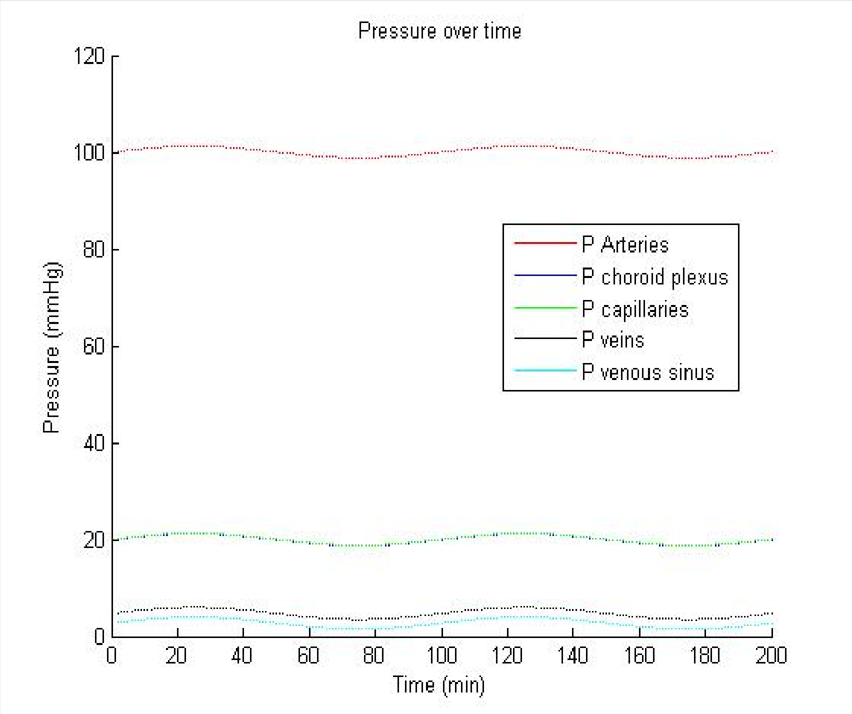
(3) Tube law:

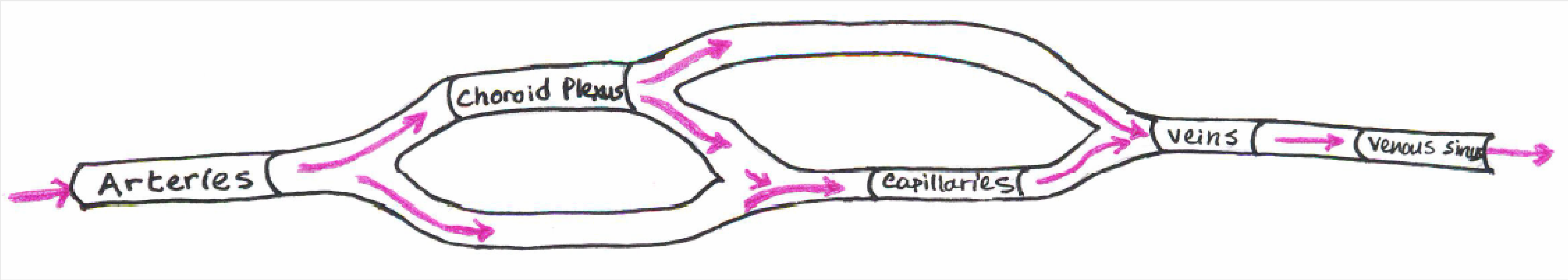
$$P_a - P_{brain} = E_L \left(\frac{A_a}{A_{ao}} - 1 \right)$$



Results
Blood Flow (dynamic)







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